* D B Y - 0 0 3 - 1 1 6 2 0 0 3 *

DBY-003-1162003

Seat No. ________

M. Sc. (Sem. II) Examination

July - 2022

Mathematics : CMT-2003

(Topology-II)

Faculty Code : 003
Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are total five questions.
(2) All questions are mandatory.
(3) Each question carries **14** marks.

1 Answer any seven of the following :

(1) Define with example : T_3-Space.

(2) Define with example : Normal Space.

(3) Define with example : Completely Regular Space.

(4) Define with example : Limit point Compact Space.

(5) State only Tietze Extension Theorem.

(6) Justify whether the real line $\mathbb{R}$ is compact or not ?

(7) Define with example : Complete metric space.

(8) Give Statement of the finite intersections property.

2 Answer any two of the following :

(1) Let X and Y be two topological spaces. Prove that $X \times Y$ is a T_2-*space* if and only if X and Y are T_2-spaces.

(2) Prove that, a topological space X is Hausdorff if and only if the set $\Delta = \{(x,x) : x \in X\}$ is a closed subset of $X \times X$.

(3) Let X be a T_1-space. Prove that, X is regular if and only if, for any point $x \in X$ and open set U containing x, there is an open set V containing x such that $\overline{V} \subseteq U$.

3 Answer any one of the following :

(1) (a) Prove that, homeomorphic image of a normal space is normal.

(b) Let X_1 and X_2 be two topological spaces. Prove that, $X_1 \times X_2$ is a completely regular space if and only if X_1 and X_2 are completely regular spaces.

(2) (a) Prove that, a regular space with countable basis is normal.

(b) State and prove, Tube Lemma.

4 Answer the following :

(1) State and prove, Extreme Value Theorem.

(2) Let X be a topological space. Prove that, X is compact if and only if for every collection C of closed sets $\{C_\alpha : \alpha \in I\}$ in X having the finite intersection property.

$$\bigcap_{\alpha \in I} C_\alpha \neq \emptyset.$$

5 Answer any two of the following :

(1) Prove that, a sequence in $\mathbb{R}^n$ is convergent relative to the Euclidean metric d if and only if it is convergent relative to the square metric ρ.

(2) Let X and Y be two topological spaces. Prove that X × Y is a compact space if and only if X and Y are compact spaces.

(3) Prove that, every compact subspaces of a metric space is closed and bounded.

(4) State and prove, Heine-Borel Theorem.